Neutron Optics and Interferometry

Dmitry Pushin

Abstract  This course is a broad introduction to the physics and applications of thermal, cold, and ultracold neutrons. Low energy neutrons have long wavelengths and interact coherently, like a wave, with solids and liquids to reveal details about their structure. Neutron scattering is an important tool for studying condensed matter, molecular, nuclear, and biological systems and neutron imaging technology is widely used by industry. Precision measurements of neutron decay and interactions are advancing the low-energy frontier of particle physics. Neutron science is an emerging field for the 21st century as several new advanced neutron sources are being constructed in the U.S., Europe, and Asia. This course will be of interest to students of nuclear physics, condensed matter physics, chemistry, biology and engineering physics. The lecture notes and homework will appear online at http://ipst.umd.edu/neutron/

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1 Lecture 1. Basic properties of neutron

1.1 Atom

The idea that matter is not infinitely divisible, but composed of fundamental entities (atoms) comes from the fifth century BC from Creek philosophers Leucippus and Democritus.

Much later (in the beginning of the 19th century) Dalton tried to systematically determine atomic weights using hydrogen atom as the basic unit. At the same time Avogadro emphasized importance to distinguish between molecules and atoms and proposed that under standard conditions, equal volumes of gases contain equal amount of molecules. That idea was a break through in determination of atomic weights. By comparing chemical properties of elements and their compounds Dmitry Mendeleev in 1871 arranged known elements of similar properties into periodic table. It was noticed that arranged number Z of an element has a value of approximate to one-half the atomic weight. That allow him to discover that some of the elements were missing (such as gallium, scandium and germanium)

With discoveries of X-rays, radioactive decay and the conduction of electricities in gases it becomes evident that atom itself has internal structure. In 1897 Thomson discovered electron with negative charge and absolute value comparable to hydrogen ion. He proposed his atomic model where neutral atom is a stable structure of the light negative electron moving through the sphere of positive electricity with mass of atom. At the beginning of the 20th century a three radiation types were discovered: (1) $\gamma$-rays which are similar to X-rays but with much shorter wavelength (Villard 1900); (2) $\beta$-rays which are identical to Thomson’s discovery of electrons (Becquerel 1900); and (3) $\alpha$-rays which are doubly ionized atoms of helium (Rutherford, Geiger, and Royds 1908-1909). Soddy in 1913 demonstrated that radioactive decay and chemical change follow by displacement law: $\alpha$-transition decreases Z by two, $\beta$-transition increases Z by one, and $\gamma$-transition leaves Z unchanged. He also named isotopes radioactive species of the same Z.

Using $\alpha$-particles to bombard atoms Geiger dismissed Thomson model of atom. In 1911 Rutherford advanced the atomic model where the mass of atom and positive charge were concentrated in central nucleus of about $10^{-14}$ m and electrons circulating in orbits of about $10^{-10}$ m in radius. As Rutherford’s model completely conflicted with established electrodynamics theory Bohr in 1913 combined his notion of stationary states with old quantum theory of Planck and Einstein and proposed his model of atom.

1.2 Neutron discovery

Following Rutherford experiments of disintegration of nitrogen by $\alpha$-particles (in the reaction $^{14}_{}N(\alpha, p)^{17}_{}O$) Bothe and Becker initiated searching 1930 for $\gamma$-
radiation emitted in \((\alpha, p)\) reactions with light nuclei. In case with Li and Be they detected a highly penetrating radiation while protons were missing. They attributed this radiation to electromagnetic nature. This conclusion was highly supported by well respected researches as Irene Curie (1931). In 1932 Webster reported that emitted radiation from Be was absorbed by matter and should have an energy of about 7 MeV which is more than twice of known \(\gamma\)-rays energies. He noted that radiation emitted in the forward direction is more energetic, thus speculating that it might be high speed corpuscles consisting of tightly bound proton and electron. As there were not detected any tracks in the cloud chamber of this corpuscles he abandoned his theory.

At the same time Curie and Joliot discovered that radiation from Be knock out protons from paraffin and attributed that to a Compton scattering of \(\gamma\)-rays on protons. Almost immediately in 1932 Perryn suggested that the mystery rays were neutrons and Majorana suggested that Joliot and Curie found neutral proton and had not recognized it.
In 1932 Chadwick (Fig. 1) trying to understand Joliot and Curie experiment performed series of experiments using apparatus schematically outlined in Fig. 2. As a $\alpha$-source he used a Po. $\alpha$-particles was bombarding Be to emit unknown highly penetrating rays. The Be emitted radiation ejected particles from hydrogen, helium, lithium, beryllium, carbon, air, and argon. The particles ejected from hydrogen behave like protons and from the other elements appeared as recoil atoms of the elements. From the conservation of the momentum and energy Chadwick shown that Be emitted radiation could not be a $\gamma$-rays (quantum radiation). Analyzing the energy relations from the studied reaction he concluded that Be emitted radiation had to be a neutral particle with mass similar to proton, which he called neutron. To determine neutron mass he bombarded B with $\alpha$-particles:

$$^{11}\text{B} + ^4\text{He} \rightarrow ^{14}\text{N} + n,$$

as masses of B and N were well known. From conservation of energy:

$$\frac{m_\alpha v_\alpha^2}{2} + m_\alpha c^2 + m_B c^2 = \frac{m_N v_N^2}{2} + m_N c^2 + m_n c^2 + m_n c^2,$$

where $v_{\alpha,N,n}$ are velocities of $\alpha$-particle, nitrogen, and neutron, while $m_{\alpha,B,N,n}$ are masses of $\alpha$-particle, boron, nitrogen, and neutron. Thus:

$$m_n c^2 \approx \frac{\frac{m_\alpha v_\alpha^2}{2} + m_\alpha c^2 + m_B c^2}{1 + \frac{v_\alpha^2}{c^2} - m_N c^2}.$$

In order to determine the speed of neutron Chadwick assumed that the neutron mass was close to the proton mass. He repeat the experiment when he bombarded hydrogen atoms with emitted neutrons to determine the speed of the protons after the collisions. By setting the neutron speed in 3 equal to those protons speeds he get the neutron mass of 938 ± 1.8 MeV. This value of the neutron mass is consisted well with currently accepted value 939.565378(21) MeV.

Later Heisenberg (1932) and Ivanenko (1932) suggested that neutron is a spin $\frac{1}{2}$ fermion and not a bound state of proton and electron. They independently proposed the nuclear model where nuclear consist of protons and neutrons. They postulated attractive interaction between proton and neutron and between neutron and neutron. These ideas were further developed by Majorana.

Note: Although more than half of the mass of the Earth and all the known planets (or almost anything you see) is due to the neutrons these particles were not discovered until they were liberated from inside the nucleus. Also enormous quantity of neutrons are continuously produced in nuclear reaction in the Sun we can not observe them due to the free neutron decay (neutron lifetime is about 15 min). To recognize Chadwick’s discovery he was awarded in 1935 the Nobel prize in physics.
1.3 Neutron detection

Any particle can be seen only because of interaction with matter. The usual and the simplest interaction is the Coulomb interaction between charged particles. Though detecting a charged particles should be simple, the detection of the individual charged particles (such as electron, proton, $\alpha$-particle, etc) was predated by the detection of another neutral particle - the photon. This is naturally happens due to our ability to detect a visible light via complex process of the photochemical reaction and conversion to neural signal. As we can see objects we have developed photographic films which are nothing more then a light quanta detectors. Later there were developed films which are sensitive to a quanta invisible to our eyes. Such detectors helped to discover high energy radiation such as X-rays and $\gamma$-rays. The more direct way to detect photons would be a photoelectric effect, where the photons knock out of electrons out of atoms. In other words in order to detect electrically neutral particle by producing of a charged one. The similar idea can be used to detect neutron.

1.3.1 Conversion to charged particle

We have already seen in Sec. 1.2 one of the examples of neutron detection via conversion (head-on collision) to a proton. The proton then strongly interacts with atoms in the gaseous medium, which becomes ionized. The charges then change the potential on the detector electrodes. This mechanism is called Geiger-Muller detector. It requires a voltage of at least 1 keV in order to prevent recombination and ensure that charges moves to electrodes fast, so the detector is ready for the next event. The crucial factor to effect ionization efficiency is the particle velocity. The lower the velocity of the neutron the longer it spends next to the nuclear in the target (probability of the neutron detection is usually follows a $1/v$ law). I.e. the most ionization takes place when the particle is about to stop. Not all kinetic energy of the particle goes to the ionization, some energy can be absorbed by nucleus as a whole, producing the excited sate that decays by emitting a $\gamma$-ray or could collided inelastically without ionization and heating up the detector.

The neutrons which we are going to talk the most ($< 10$ eV) are too slow to cause ionization. The mechanism explode in slow neutron detection consist of a nuclear reaction followed by emission of a proton or $\alpha$-particle. The typical nuclear reactions used here are:

\begin{align*}
3^\text{He} + n &\rightarrow T + p + 0.765 \text{ MeV} \\
6^\text{Li} + n &\rightarrow T + ^3\text{He} + 4.78 \text{ MeV} \\
10^\text{B} + n &\rightarrow ^7\text{Li} + ^4\text{He} + 2.79 \text{ MeV} \\
10^\text{B} + n &\rightarrow ^7\text{Li}^* + ^4\text{He} + 2.31 \text{ MeV} \\
^7\text{Li}^* &\rightarrow ^7\text{Li} + \gamma + 480 \text{ keV} .
\end{align*}

(4)
1.3.2 Gaseous proportional counters

The first and last reactions of the Eq. 4 are the most frequently used in the slow neutron gaseous detectors, which are filled either with enriched $^3\text{He}$ or with the BF$_3$ enriched in $^{10}\text{B}$. The detectors usually have a cylindrical dorm with a thin wire (anode) stretched along the axis as shown in Fig. 3. The efficiency of such a detector depends on the gas pressure, the detector length, and the probability of the neutron absorption. The efficiency of such detectors could be made close to 100%. Gaseous detectors could be modified in a such way that in addition to the particle registration they can show the place at which particle arrived. Such detector usually called position sensitive detectors.

![Fig. 3 Schematic diagram of a gaseous detector](image)

1.3.3 Fission chambers

The detectors which uses neutron capture induced fission of the radioactive elements called fission chambers. Such elements are $^{233}\text{U}$, $^{235}\text{U}$, $^{237}\text{Np}$, and $^{239}\text{Pu}$. Such detector usually consist of a thin layer of fissile material (usually coated on the walls of the ionization chamber). The thickness of a such material is a very important factor as these materials are natural $\alpha$ emitters. Thus such detector should be able to discriminate between signal coming from ionization due to the $\alpha$-particles and from
the fission fragments. For the same reason the volume of ionization gas should be small. Due to these factors these detectors usually have the low efficiency and often used as a beam monitors.

### 1.3.4 Scintillation detectors

Neutrons can be also detected via scintillation, where neutron absorption is followed by emission of an energetic charged particle, which ionized the working medium and causes fluorescent radiation. The visible light quanta created in this process then enter either photomultiplier tube, photodetector, or photosensitive film.

### 1.4 Neutron properties

#### 1.4.1 Why neutrons?

Almost everything we know about structure of the matter we know from scattering experiments. These usually are scattering of photons, electrons, protons, neutrons, atoms, etc. The knowledge we acquire is not only chemical structure and physical

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**Fig. 4** Neutron and X-ray interaction with different elements
arrangements of the atoms/molecules but also dynamical and internal properties of atoms, nuclei, etc. Very often these scattering techniques are complimentary due to uniqueness of interaction and there energies. In todays world, when we are looking for new materials experimental methods employing neutrons and X-rays are the techniques of choice. The slow neutron techniques are so unique and hardly matched by other methods. Some of such properties are:

- Neutrons interact with nuclei and not with electrons as photons, electrons, and protons do. The consequence of that neutrons interact stronger then X-ray with lighter atoms. Figure 4 compares X-ray and neutron cross sections for different atoms. Due to the same reasons neutrons are highly penetrating into the bulk and thus good for studying matter at extrem conditions.
- For similar wavelength neutron energy are smaller and comparable to elementary excitation in matter (good for determining phonon spectra).
- Neutrons have magnetic moment, thus good to study magnetic properties of matter.
- Neutrons have mass and slow enough to use them in studies of fundamental physics.

<table>
<thead>
<tr>
<th>Energy range</th>
<th>Classification nuclear physics</th>
<th>Classification neutron scattering</th>
<th>Energy range neutron scattering</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 1 keV</td>
<td>slow</td>
<td>ultra cold</td>
<td>&lt; 0.1 meV</td>
</tr>
<tr>
<td></td>
<td></td>
<td>very cold</td>
<td>0.1 ÷ 0.5 meV</td>
</tr>
<tr>
<td></td>
<td></td>
<td>cold</td>
<td>0.5 ÷ 5 meV</td>
</tr>
<tr>
<td></td>
<td></td>
<td>thermal</td>
<td>5 ÷ 100 meV</td>
</tr>
<tr>
<td></td>
<td></td>
<td>epithermal or hot resonant</td>
<td>0.1 ÷ 1 eV</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 ÷ 100 eV</td>
</tr>
<tr>
<td>1 keV ÷ 0.5 MeV</td>
<td>intermediate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5 ÷ 10 MeV</td>
<td>fast</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 ÷ 50 MeV</td>
<td>very fast</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05 ÷ 10 GeV</td>
<td>high energy or ultra fast</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&gt; 10 GeV</td>
<td>relativistic</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 5** Approximate list of neutron energy regimes classified by names (from Furrer’s book)

The energy of the neutrons span many orders of magnitude. Figure 5 gives approximate classification of the neutrons terms for different energy ranges. It also compares with nuclear physics classification.

### 1.4.2 Neutron wavelength

A free slow neutron moving with velocity $v \ (v << c)$ has the Kinetic energy
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\[ E = \frac{m_n v^2}{2}, \]  
\( (5) \)

where \( m_n \) is neutron mass. We can assign to such neutron the deBroglie wavelength:

\[ \lambda = \frac{\hbar}{m_n v}, \]  
\( (6) \)

where \( \hbar = 6.626 \times 10^{-34} \text{ J s} \) is the Planck constant. The wave vector of the neutron has magnitude:

\[ k = \frac{2 \pi}{\lambda}. \]  
\( (7) \)

We can combine these definitions as: We can assign to such neutron the deBroglie wavelength:

\[ E = \frac{h^2}{2m_n \lambda^2} = \frac{h^2 k^2}{2m_n} = \frac{m_n v^2}{2} = k_B T, \]  
\( (8) \)

where \( h = \frac{\hbar}{2\pi} \) and \( k_B = 1.381 \times 10^{-23} \text{ J K}^{-1} \) is the Boltzmann constant. By substituting values of the constant and using units meV for \( E \), Å for \( \lambda \), Å\(^{-1}\) for \( k \), km/s for \( v \), and K for \( T \) we have

\[ E = 81.81 \cdot \frac{1}{\lambda^2} = 2.072 \cdot k^2 = 5.227 \cdot v^2 = 0.08617 \cdot T. \]  
\( (9) \)

**Table 1.1** Properties of the neutron (Taylor 1990)

<table>
<thead>
<tr>
<th>Particle properties</th>
<th>Connection</th>
<th>Wave properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m = 1.674928(1) \times 10^{-27} \text{ kg} )</td>
<td>de Broglie</td>
<td>( \lambda_c = \frac{\hbar}{mc} = 1.319695(20) \times 10^{-15} \text{ m} )</td>
</tr>
<tr>
<td>( \tau = \frac{1}{\hbar} )</td>
<td>(thermal neutrons: ( \lambda = 1.8 \text{ Å}, v = 2200 \text{ m/s} ))</td>
<td></td>
</tr>
<tr>
<td>( \mu = \mu_B = -9.6623755(71) \times 10^{-27} \text{ J/T} )</td>
<td>Schrödinger</td>
<td>( \lambda_b = \frac{\hbar}{m_B} = 1.8 \times 10^{-10} \text{ m} )</td>
</tr>
<tr>
<td>( R = 0.7 \text{ Å} )</td>
<td>( \hat{H}\psi(r,t) = i\hbar \frac{\partial \psi(r,t)}{\partial t} )</td>
<td>( \Delta_{\psi} = \frac{1}{2\Delta k} \leq 10^{-4} \text{ m} )</td>
</tr>
<tr>
<td>( a = 12.0(2.5) \times 10^{-4} \text{ Å} )</td>
<td>( a_{\text{d-d quark structure}} )</td>
<td>( \Delta_{\psi} = v\Delta t \geq 10^{-2} \text{ m} )</td>
</tr>
<tr>
<td>( u-d \text{ quark structure} )</td>
<td>0 ( \leq \chi \leq 2\pi/4\tau )</td>
<td>( \Delta_{\psi} = v\tau = 1.942(5) \times 10^{10} \text{ m} )</td>
</tr>
</tbody>
</table>

\( m: \text{ mass} \)
\( \tau: \text{ spin} \)
\( \mu: \text{ magnetic moment} \)
\( \hat{\lambda} = -1.919040758(45) \times 10^{-27} \text{ J/T} \), magnetic moment in units of the nuclear magneton
\( \mu_\text{N} = 5.05078558(30) \times 10^{-27} \text{ J/T} \), nuclear magneton
\( \tau: \beta\text{-decay lifetime} \)
\( R: \text{ (magnetic) confinement radius} \)
\( ac: \text{ electric polarizability} \)
All other quantities like electric charge, magnetic monopole and electric dipole moment are compatible with zero

**Fig. 6** Table of neutron properties
For a photon: \( E \text{ (meV)} = 0.124/\lambda \text{ (cm)} \)

\[
E = \frac{\hbar^2 k^2}{2m} = \frac{k^2}{2m\lambda^2} = hf = \frac{1}{2} mv^2 = \frac{1}{2} m(t/d)^2 = k_B T
\]

\[
E = 2.072 k^2 = \frac{81.79}{\lambda^2} = 4.135 f = 5.228 \times 10^{-6} v^2
\]

\[
= 5.228 \times 10^6 (t/d)^{-2} = 0.0862 T
\]

\( E \text{ (meV)}, \lambda \text{ (Å)}, f \text{ (THz)}, v \text{ (m s}^{-1}), k \text{ (Å}^{-1}), t \text{ (μs m}^{-1}), T \text{ (K)} \)

For a 1 Å neutron:
\( E = 81.79 \text{ meV}, T = 949 \text{ K}, v = 3955 \text{ m s}^{-1} \)
\( f = 19.78 \text{ THz}, k = 6.283 \text{ Å}^{-1}, t/d = 253 \text{ μs m}^{-1} \)

Fig. 7 Conversion Table (taken from Sam Werner paper)

There is an easy was to remember some of relations via Sam Werner’s rule of “the Neutron twos”:

\[
E = 20 \text{ meV} \rightarrow \lambda \approx 2 \text{ Å} \rightarrow v \approx 2000 \text{ m/s}.
\]

A fee neutron half-life \((T_{1/2}\) is about 10.3 minutes, combining it with neutron velocity:

\[
T_{1/2}v = 618 s \cdot 2000 \text{ m/s} = 1,236 km.
\]